FREQUENCY DOMAIN ELECTROMAGNETIC INVESTIGATION ON ELONGATED CONDUCTIVITY STRUCTURES

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Abstract
After a short overview of the geophysical methods applied in geothermal exploration emphasis is put on frequency domain EM methods because they are widely used in geothermal projects. The main features of magnetotellurics (MT) and controlled source audiomagnetotellurics (CSAMT) are provided and the partial differential equations describing the natural and controlled source method are also given for 2D situation. Finite difference numerical model results are also presented for 2.5D situation. It can be stated that in the course of interpretation the use of normalized impedance sections over conductivity inhomogeneities can be prosperous if the geology is far from being complex.

1. Introduction

In order to decrease geological risk in geothermal exploration and optimize the production of geothermal energy, a wide range of geophysical methods is applied in geothermal projects. There is no general recipe for the selection of the geophysical methods, however some similarities can be found between the methods and their sequence as well used in geothermal and oil exploration and also between the geophysical logs in geothermal and oil wells. At the same time there are differences as well, because after the application of geophysical methods with less resolution, emphasis is usually put on 3D seismic method in CH exploration, while in geothermal exploration this method has fewer applications in percentage due to financial reason, too. The presence of a geothermal reservoir usually results in resistivity decrease. For this reason depending on the depth and the size of the geothermal reservoir the use of different geoelectric and electromagnetic methods can be prosperous in geothermal exploration. The shape of the geothermal reservoir can usually be characterized by 3D structures. However sometimes the geometry of a reservoir can be approximated by elongated structure, where the direction of elongation coincides with the structural strike direction.

2. Geophysical methods used in geothermal exploration

Besides geological and geochemical observations the geophysical investigations must be mentioned as organic parts of the geothermal exploration. The primary aims of these surface observations are to obtain initial estimates on the size of the reservoir and the geological structure of the geothermal system. The geothermal fields occur in different rock types and generally situate in convection systems in which usually hot water rises from great depth and is trapped in the reservoirs with cap rock. For this reason the primary objective of geothermal exploration is not the identification of the rock-type of the reservoirs, but rather the localization of tectonic elements such as faults or fractures along which high heat flow can be expected.

The greatest heat flow can be experienced over geothermal fields connected to mantle plumes, rift or subduction zones. Significantly less, but greater than average heat flow can
be observed in areas where sedimentary rocks with great thickness cover fractured or faulted basement rock and the thickness of the crust is relatively small. The exploration of the geothermal system with the greatest heat flow requires the set of geophysical methods different from the one which is applied for the geothermal exploration in sedimentary basin.

The most important methods in the course of geothermal exploration are gravity, magnetic, subsurface temperature, electrical, electromagnetic and seismic method. Summaries on the geophysical methods applied in geothermal exploration were presented by [1–3].

The gravity method investigates the spatial variation of the gravity field usually on the surface. The result of gravity survey is the Bouguer anomaly map. This map is obtained after normal, free-air, Bouguer, topographic (and for airborne and marine survey Eötvös) corrections are applied to gravity field data. It is sensitive to the lateral changes in density, for this reason it can be suitable to map bedrock topography and to locate faults. By means of filtering the depth interval for the origin of gravity information can be controlled. Gravity monitoring survey (repeated gravity measurement on the same stations) can be an effective tool to observe the fluid extraction and fluid recharge in time. It can provide information on the need of reinjection too.

The spatial distribution of the magnetic field is determined by magnetic method. Airborne and ground surveys can be distinguished. Normal-field correction (taking into account the normal variation of the geomagnetic field in the function of latitude and longitude) and diurnal correction (to account for the temporal variations of the geomagnetic field during the measurement) have to be made to obtain magnetic anomaly map showing the presence of magnetic bodies. The resultant magnetization of a magnetic body is the vectorial sum of the remanent and the induced magnetization. Both depend on the type of the ferromagnetic minerals and their concentration in the rocks. The most common ferromagnetic minerals are titanomagnetite and magnetite. One of the most important applications of magnetic method is mapping near-surface effusive and intrusive rocks. The greater the depth of the magnetic body with fixed magnetization and shape is, the less its magnetic response and the longer of the wavelength of the response is. The pattern of the magnetic anomaly is influenced by the direction of the inducing magnetic field. To make magnetic data interpretation easier, reduction to pole process (the magnetic anomaly map is recomputed for the case as if it had a vertical inducing field) is applied to magnetic field data. The simultaneous use of gravity and magnetic map over sedimentary basin can provide information on the types of rock building up the basement complex. Under the depth of Curie point there are no minerals with ferromagnetic properties. From the magnetic map covering a large area the depth to the Curie point isotherm can be estimated as well.

Subsurface temperature measurements can be considered as the most direct survey for detecting and investigating area of anomalous heat flux. Earlier temperature measurements were made only in holes with a few meters of depth, nowadays temperature measurements are recommended in wells with depth of 100m at least. Even temperature measurements in shallow hole can indicate the presence of fractures along which hot water is rising up. However thermal gradient surveys at depth where the effect of the annual temperature change can be negligible are more informative. In the knowledge of thermal conductivity and geothermal gradient the heat flow along the direction of borehole can be determined based on the first Fourier’s law.

Electrical and electromagnetic methods are widely used in geothermal exploration, because the physical parameters measured by these methods are sensitive to the resistivity
Frequency Domain Electromagnetic Investigation on Elongated Conductivity Structures

of the formations to be investigated. These methods can be characterized with greater spatial resolution than gravity or magnetic methods. Besides information on depth, shape and size these methods may be efficiently applied to locate fractured zones filled with fluid. The higher the temperature and the salinity of the water is, the easier is to detect the presence of water. Electrical conductivity is also affected by porosity and water saturation. Conductivity increases with increasing porosity and increasing water saturation. The detectability of the geothermal zone depends on its depth and size and the resistivity contrast between the geothermal zone and its surroundings. These geophysical methods can be classified on the basis of their source origin. In this aspect natural and controlled source surveys can be distinguished. Self potential (SP) and magnetotelluric (MT) methods belong to the group of natural source methods. Direct current resistivity, controlled source audio magnetotelluric (CSAMT) and transient electromagnetic (or time domain electromagnetic, TEM) methods are the most important ones among controlled source surveys applied in geothermal projects. Additional classification can be made on the frequency. Geoelectrical methods are DC methods, where the exploration depth can be controlled by the alteration of the measuring array. The electromagnetic methods utilize usually a range of frequency values depending on the exploration depth. The controlled source methods can work in frequency and in time domain. The EM fields of controlled source methods can be generated by inductive or conductive way. Besides frequency range it is the transmitter-receiver distance which has to be adjusted to the exploration depth.

Seismic methods applied in geothermal exploration can be passive or active. Geothermal fields are frequently characterized by increased micro earthquake activity. In the course of the exploration period passive seismic method can be used if there are seismically active fracture zones in the prospect area. To obtain sufficient number of observations the microseismic surveys may take some weeks. The objective of the survey is to delineate the anomalous area of Poisson’s ratio. Besides the spatial identification of each earthquake centre the determination of compressional and shear wave velocities (from which the Poisson’ ratio can be concluded) is also needed. The calculated Poisson’s ratio values are referred along the straight line travel paths between epicenter and seismograph. The zones with increased Poisson’s ratio can be correlated to the part of the reservoir with high permeability. Another application of microseismicity can be to monitor fluid injection into the reservoir (or extraction from it) resulting in induced seismicity. Active seismic methods apply artificial sources (i.e. explosive charge or vibroseis). Depending on the geology seismic refraction or seismic reflection survey is used. The physical condition of developing seismic refracted wave (head wave) is the increasing velocity with depth. The aim of this survey is to determine the relief of the refractor(s). If we make a comparison between exploration geophysical methods, it can be stated that seismic reflection method can be characterized with the highest resolution. This is the most expensive (geophysical) exploration method. The physical condition of having elastic wave reflection from an interface is that acoustic impedance difference should be between the formations in contact. The objective of seismic reflection survey is to map subsurface interfaces (including the depth, dip and strike of bedding and lateral changes in the reflectors) and to define stratigraphic variations as well. In geothermal exploration detailed seismic survey is recommended to map fault systems.

Geophysical measurements are also carried out in boreholes. The set of these measurements are called well logging. It is not included in the geophysical exploration methods (because they are considered as surface methods). The purpose of the open hole measurements can be the in-situ determination of the important physical parameters as
porosity, permeability, temperature and to provide technical parameters for drilling engineers (i.e. caliper log for cementing). Additional parameters and indirectly technical assistance are provided by production logging and logging aiming at well inspection.

3. Frequency domain electromagnetic (FEM) methods

In the magnetotelluric (MT) method natural EM fields with variable direction, frequency and strength are used to investigate the resistivity distribution of the earth [4]. Sources of MT fields above 1Hz are thunderstorms, from which lightning radiates EM fields propagating at great distances. Sources of MT fields below 1Hz are EM fields due to the interaction between current systems in magnetosphere and the solar activity. Independent of the natural EM field’s origin these fields can be considered at the surface of the earth as plane waves. Only a small amount of their energy penetrates and propagating vertically downward into the earth. The attenuation of the vertically propagating EM fields depends on the subsurface conductivity and the frequency. At interfaces some part of the energy is reflected back to the surface, while the other part of the EM fields is propagating downward after refraction. At the surface the superposition of the incident (primary) and the reflected EM fields can be measured. The ratio of electric and magnetic field components perpendicular to each other is the impedance which is used to define apparent resistivity. In exploration mainly the apparent resistivity frequency sounding curve and its phase frequency sounding curve are applied. The latter is the same as the phase frequency sounding curve of the impedance. In the interpretation of sophisticated geology the tipper and the polar diagrams of impedance tensor elements are used. Over geological situation complicated than horizontally stratified half-space the impedance is not scalar quantity. For elongated and arbitrary (3D) situation the polar diagrams for the elements of impedance tensor are derived from the measurements usually at many frequencies, because the strike direction may change with depth. Except for near lateral conductivity changes the vertical magnetic field component can usually be assumed zero. The relation between the vertical and horizontal magnetic field components at any frequency can be expressed by means of tipper. The main advantage of the MT method is that exploration from shallow to very great depth (including lower crust) can be made. The depth of investigation of MT is much greater than in case of any controlled-source method. The shortcomings of the MT method are the difficulty of data acquisition in electrically noisy areas and the time needed for MT measurements to obtain data at all frequencies.

To overcome the natural signal strength problem in MT and to control the source polarization the controlled source audio-frequency magnetotellurics (CSAMT) with grounded electric dipole source was proposed by [5]. Similarly to magnetotellurics, the orthogonal electric and magnetic components are measured and the plane wave approximation is used to calculate scalar apparent resistivity. In general, controlled source frequency domain electromagnetic (FEM) methods apply either magnetic or electric dipole sources [6]. In both cases they can work in different zones defined by the product of wave number and transmitter-receiver separation. From the interpretation point of view the ideal situation is to measure in the far-field regime for all frequencies at any transmitter-receiver separation (r). Assuming horizontal electric dipole (HED) source field (just like in case of magnetic dipole source) we can separate three zones over the surface of the homogeneous half-space. In the far field zone the horizontal E, H components decay as $1/r$ and the penetration depth depends on frequency and resistivity. It is also called plane-wave zone,
because the basic resistivity MT definition can be applied. The next zone is the transition zone in which the electric field varies as $1/r^3$ and the magnetic field decays as $1/r^2$ to $1/r^3$ and both depend upon frequency. The third zone is the near-field zone, in which the horizontal $E$ and $H$ are independent of the frequency, $E$ decays as $1/r^3$ and $H$ as $1/r^2$. In the near-field zone the penetration depth can be controlled only by the geometry of the EM array opposite to the transition zone, where besides the geometry the frequency is also an influencing factor. For homogeneous earth the far field assumption fulfills when $r_{\text{min}} > 6\delta$, where $\delta$ denotes skin depth. Measurements are made in the transition zone if they are carried out at smaller frequencies or with less separation characterizing the far field zone and $r_{\text{min}} > 0.2\delta$. These zone limits were given by [7]. In these two regimes the EM fields depend on frequency and resistivity. However, decreasing frequency information can be obtained with approaching near field zone. The main advantages of CSAMT method over MT are a better signal to noise ratio, and the control of source polarization with respect structural strike. At the same time it has to be noted that certain source effects can cause some difficulties in the interpretation of controlled source EM data.

Both methods have a lot of geothermal applications all over the world. The increasing number of MT and CSAMT applications is proved by papers as well. [8] presented electromagnetic studies in geothermal regions among the first. Mainly audio MT is preferred by [2]. A lot of papers in this subject can be found in Geophysics and Geothermics. [9] showed the advantage of the combination of these two methods for a geothermal area in New Mexico. Special issue was presented by Elsevier in the Journal of Applied Geophysics with the title Electrical and Electromagnetic Studies in Geothermally Active Regions in 2006. In [10] emphasis is put on EM imaging of geoelectric structures of geothermal fields and some new developments in EM methods of geothermal exploration are also given.

4. Partial differential equations of FEM for elongated conductivity structures

If the investigated conductivity structure is elongated and the source field is EM plane wave, the problem to be solved is a pure 2D one. If the EM source which can be treated mathematically as a point source, we encounter a 2D model exited by a 3D source. This situation is called a 2.5D problem. For these two situations the partial differential equations are presented and the similarities and differences between them are also given. In the course of EM investigation on elongated conductivity structures exited by electric dipole source special emphasis has to be put on two modes. Distinction is made between TE mode, when the electric source field is parallel to the structural strike, and TM mode, when the electric source field is perpendicular to it.

In MT and CSAMT equations $e^{i\omega t}$ time dependent EM field variation is assumed and let $x$ coincide with the strike direction. The basic relationships governing the electromagnetic phenomenon are the Maxwell’s equations. There is no difference in the form of the next Maxwell’s equation, i.e.:

$$\text{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$  \hspace{1cm} (1)
\( \hat{\delta} = \Im \delta (\vec{r}) \) makes possible a point source approximation. This source term equals zero for MT on the right side of the following equation, which is the other Maxwell’s equation:

\[
\text{rot} \vec{H} = \vec{j} + \hat{\delta}
\]  

(2)

Taking into account that the partial derivative of the EM components with respect \( x \) equals zero for MT, the component equations of (1) and (2) can be written as:

\[
\left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) = -i\sigma \mu E_x \quad (3)
\]

\[
\frac{\partial E_x}{\partial z} = -i\sigma \mu E_y \quad (4)
\]

\[
\frac{\partial E_x}{\partial y} = i\sigma \mu E_z \quad (5)
\]

\[
\left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) = \sigma E_x \quad (6)
\]

\[
\frac{\partial H_x}{\partial z} = \sigma E_y \quad (7)
\]

\[
-\frac{\partial H_x}{\partial y} = \sigma E_z \quad (8)
\]

If equations (4), (5), (6) are taken into one set of equations, only \( E_x, H_y, H_z \) components, and similarly in (3), (7), (8) only \( H_x, E_y, E_z \) can be found. The conclusion is as follows: in case of 2D situation independently of the angle of incident the plane waves the EM fields can be divided into two parts, which can be treated separately. The first mode is called TE mode, the second is the TM mode. To present the 2D MT equations for homogeneous case in the first group the value of magnetic field component from equations (4), (5) into equation (6) and in the second group the value of electric field component from equations (7), (8) into equation (3) have to be substituted. In this way the most comprehensive form for the TE mode is:

\[
\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = i\sigma \mu \epsilon E_x \quad (9)
\]

After the operations mentioned above for the TM mode a similar differential equation can be derived:
Frequency Domain Electromagnetic Investigation on Elongated Conductivity Structures

\[ \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = i \omega \mu \sigma H_x \quad (10) \]

These equations can be solved with either finite difference (FD) or finite element (FE) method. In spite of the formal similarity of the TE and TM mode the solutions of the two modes are different due to the inner boundary conditions.

In case of magnetic source [11] recommended that the solution of the original 3D problem could be replaced by the series of solutions of 2D problem in the \( k_z \) wave number domain. To follow this procedure the Maxwell’s equations (1) and (2) are Fourier-transformed. The way of transformation can be expressed as:

\[ \mathcal{F}(k_x, y, z) = \int_{-\infty}^{\infty} \mathcal{F}(x, y, z) e^{-ik_x x} dx \quad (11) \]

In course of the transformation the Fourier transform of the partial derivative of the EM field components with respect \( x \) – which is proportional the Fourier transform of the component itself – is also needed.

Opposite MT assuming source term in equation (2) for the Fourier transforms of the Maxwell’s component equation can be written as:

\[ \partial \tilde{E}_z / \partial y - \partial \tilde{E}_y / \partial z = -i \sigma \mu \tilde{H}_x \quad (12) \]
\[ \partial \tilde{E}_x / \partial z - i k_x \tilde{E}_z = -i \sigma \mu \tilde{H}_y \quad (13) \]
\[ i k_x \tilde{E}_y - \partial \tilde{E}_x / \partial y = -i \sigma \mu \tilde{H}_z \quad (14) \]
\[ \partial \tilde{H}_x / \partial y - \partial \tilde{H}_y / \partial z = \sigma \tilde{E}_x + i k_x \quad (15) \]
\[ \partial \tilde{H}_x / \partial z - i k_x \tilde{H}_z = \sigma \tilde{E}_y + i k_y \quad (16) \]
\[ i k_x \tilde{H}_y - \partial \tilde{H}_x / \partial y = \sigma \tilde{E}_z \quad (17) \]

Equation (12) is the only one which resembles MT TM mode and equation (15) resembles MT TE mode. As the other equations have EM fields components belonging to both modes, for this reason the two modes cannot be separated. Here TE mode denotes the situation when the direction of electric dipole source is parallel to the strike, and TM mode corresponds to the case when it is perpendicular to it as it can be seen in Fig.1. In the next the most comprehensive form for the two modes will be given. The aim of this is that instead of having three EM components, it is more comfortable to work with two EM components for each mode. Introducing the following notation as \( k^2 = -i \sigma \mu \sigma \),
\[ \zeta^M = \sigma \left( 1 - \frac{k_x^2}{k^2} \right), \quad \zeta^E = i \sigma \mu \left( 1 - \frac{k_x^2}{k^2} \right) \quad \text{a partial differential equation} \]

system can be derived for the two polarizations. From equations (12)-(17), without going into details for TE mode:

\[ -\frac{\partial}{\partial y} \left( \frac{1}{\zeta^M} \frac{\partial \tilde{H}_x}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{1}{\zeta^M} \frac{\partial \tilde{H}_x}{\partial z} \right) - ik_x \frac{\partial \tilde{E}_x}{\partial y} + ik_x \frac{\partial \tilde{E}_x}{\partial z} + \nu^M \tilde{H}_x = 0 \quad (18) \]

\[ -\frac{\partial}{\partial y} \left( \frac{1}{\zeta^E} \frac{\partial \tilde{E}_x}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{1}{\zeta^E} \frac{\partial \tilde{E}_x}{\partial z} \right) + ik_x \frac{\partial \tilde{H}_x}{\partial y} - ik_x \frac{\partial \tilde{H}_x}{\partial z} + \nu^E \tilde{E}_x = -\tilde{h}_x \quad (19) \]

Similar coupled differential equations can be written for TM mode:

\[ -\frac{\partial}{\partial y} \left( \frac{1}{\zeta^M} \frac{\partial \tilde{H}_x}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{1}{\zeta^M} \frac{\partial \tilde{H}_x}{\partial z} \right) - ik_x \frac{\partial \tilde{E}_x}{\partial y} + ik_x \frac{\partial \tilde{E}_x}{\partial z} + \nu^M \tilde{H}_x = -\frac{\partial}{\partial z} \left( \frac{\tilde{b}_y}{\zeta^M} \right) \quad (20) \]

\[ -\frac{\partial}{\partial y} \left( \frac{1}{\zeta^E} \frac{\partial \tilde{E}_x}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{1}{\zeta^E} \frac{\partial \tilde{E}_x}{\partial z} \right) + ik_x \frac{\partial \tilde{H}_x}{\partial y} - ik_x \frac{\partial \tilde{H}_x}{\partial z} + \nu^E \tilde{E}_x = ik_x \frac{\partial}{\partial y} \left( \frac{\tilde{E}_y}{\zeta^E} \right) \quad (21) \]

In these equations the value of \( \zeta = \left( \frac{k_x^2}{k^2} - 1 \right)^{-1} \). Just like in case of MT either FD or FE method can be applied, however the 2D problem has to be solved for a lot of discrete wave number values here. In the knowledge of strike directional field components in function of wave number inverse Fourier transformation has to be applied to gain spatial solution. In order to get rid of the effect of transmitter-receiver distance on the EM field components the use of impedance can be recommended. For example in TE mode it can be determined as:

\[ |Z_{TE}| = \left| \frac{E_x}{H_y} \right| = \int_0^{k_{x,\max}} \tilde{E}_x dk_x \]

\[ \left| \tilde{E}_x \right| \left| \tilde{H}_y \right| \left( \int_0^{k_{x,\max}} \frac{\partial \tilde{E}_x}{\partial z} dk_x - \frac{\partial \tilde{H}_x}{\partial y} \right) \quad (22) \]

It is more complicated than in MT, where it can be calculated as:

\[ |Z_{xy}| = \left| \frac{E_x}{H_y} \right| = \frac{\sigma \mu |E_x|}{\left| \frac{\partial E_x}{\partial z} \right|} \quad (23) \]
Frequency Domain Electromagnetic Investigation on Elongated Conductivity Structures

Figure 1: TE and TM mode for controlled source FEM and the investigated model

Figure 2: Impedance amplitude sections for TE and TM mode
5. Results of numerical modeling of 2.5D FEM

In the next paragraph some results of numerical modeling are presented. Differential equations of the 3. paragraph were finite differenced and block-tridiagonal LU decomposition was used to solve the resulting linear set of equations for suitable number of wave numbers. To get spatial solution numerical inverse Fourier transformation was applied.

![Figure 3: Impedance phase sections for TE and TM mode](image)

In the upper part of Figure 2 and Figure 3 the impedance amplitude and phase distribution is given for the conductivity structure of Figure 1. The horizontal axis is the transmitter-receiver distance, frequency is plotted along the vertical axis of the sections. In order to enhance the response of conductive inhomogeneity, normalized impedance amplitude and normalized impedance phase sections are determined for TE (T_x–R_x array in Figure 1) and TM (T_y–R_y array in the same figure) mode. The EM responses over the two-layer half-space of Figure 1, were taken as reference values for the normalization. The result of normalization can be seen in the lower part of Figure 2 and Figure 3. The method seems to be promising, however, if there is conductivity inhomogeneity under the source, the effect of source overprint can suppress the response due to the inhomogeneity under the
receiver [12]. Shadow effect also can be recognized in these situations. The shadow effect for the hosted conductive body under the receiver develops because of the stronger attenuation of the diffusive EM fields in the conductive inhomogeneity compared with its surroundings. On the basis of impedance amplitude and impedance phase sections horizontal delineation can not yield good result for the side of the elongated block away from the source (in this example at 3700m). Here the shadow effect distorts the frequency sounding curves over the side far away from the transmitter. Other geophysical methods (without source effect) or using transmitter from the opposite side may help in the interpretation.

6. Summary

The application of geophysical method cannot be abandoned in the course of geothermal exploration. After reviewing the geophysical methods used in geothermal projects the main features of MT and CSAMT were provided. Basic formalism was given for elongated conductivity structures. Results of numerical forward modeling for 2.5D FEM were also presented. Inversion should be the exact solution to determine the shape, size, depth and the conductivity of the inhomogeneity within the host rocks. However, for a relatively not sophisticated conductivity situation the normalized impedance amplitude and normalized impedance phase sections can be suggested for the two modes as a simple way of inhomogeneity imaging. The method seems to be promising in the shortage of source effects.

List of Symbols

- $E$: the electric field vector (V/m)
- $H$: the magnetic field vector (A/m)
- $B$: the magnetix flux density (Wb/m²)
- $\varepsilon$: the dielectric constant (F/m)
- $\sigma$: the conductivity (mho/m or S/m)
- $\mu$: the magnetic permeability (H/m)
- $r$: transmitter-receiver separation, distance (m)
- $\delta$: skin depth (m)
- $t$: time (s)
- $\omega$: the angular frequency of the EM field (Hz)
- $I$: transmitter current (A)
- $d\bar{s}$: elementary dipole length (m)
- $\delta(r)$: Dirac-delta function
- $i$: $\sqrt{-1}$
- $j$: the current density vector (A/m²)
- $j_s$: the applied current source (A/m²)
- $F$: EM component (V/m or A/m)
- $k$: wave number in the strike direction (1/m)
- $F$: the Fourier-transform of the EM component (V/m or A/m)
- $k$: the complex wave number (1/m)
- $\xi^T$: TM impedance (Hm⁻¹s⁻¹)
- $\xi^E$: TE impedance (Hm⁻¹s⁻¹)
- $\nu^T$: TM admittance (Hm⁻¹s⁻¹)
- $\nu^E$: TE admittance (S/m)
- $\xi$: derived parameter (m²)
- $Z$: impedance (ohm)
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