DETERMINATION OF THE TEMPERATURE DISTRIBUTION IN THE CIRCULATING DRILLING FLUID

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Abstract
An analytical mathematical model is developed to calculate the temperature distribution of the circulating drilling fluid in the drillpipe and the annulus. The obtained formulas are valid for forward circulation. Influence of geothermal, geometrical and performance variables are investigated.

1. Introduction

The knowledge of the temperature distribution in the circulating drilling fluid has a great importance to design drilling operations. Determination of the maximum allowable pumping rate, design of cementing program, calculation of thermal stresses in casing strings, logging tool design need to modeling the borehole heat transfer process. It is especially necessary for drilling geothermal wells, where significant heat transfer occurs. Fluid temperature is influenced by many variables: the depth, the elapsed time, mass flow rate and material properties of the drilling fluid, thermal properties and temperature distribution of the surrounding rocks. Although direct temperature measurements can be made at a given depth and time within drilling operations, without the heat transfer model we cannot predict temperatures for arbitrary performance states. Flow and heat transfer in drilling operations is a complex simultaneous interaction between the circulating flow and the surrounding rock mass around the borehole. The temperature of the flowing fluid in the drill pipe and the annulus is lower than the undisturbed rock temperature. This temperature-inhomogeneity induces a radial heat flux toward the borehole. It must be noted that the upper section of the borehole the fluid is warmer than the rock, thus the heat flux is directed radially outward.

The drilling fluid is heated by the surrounding rock which is cooled. The heat transfer process is a time-depending phenomenon during the drilling history. An analytical mathematical model is elaborated to describe this transient heat transfer. It makes possible to calculate temperature profiles for entire wellbore and to investigate the influence of the main performance parameters.

Two types of mathematical models, analytical and numerical, have emerged for determination the circulating fluid temperature. Their theoretical bases are the same: the balance equations of mass, momentum and energy. The initial and boundary conditions, the material properties are obviously the same. The definitive difference between the two models is the different ways to solve the equations. Applying analytical solutions the obtained differential equations are integrated in closed form as mathematical analytic functions. In order to easy integration some simplifying assumptions can be made. Analytic functions can give information about the system behavior even for lack of detailed calculations. Numerical models use some time and space stepping procedure, the solution is obtained by a generated table or a graph. Numerical techniques are suitable to consider more realistic models of greater complexity, at the cost of extended amount of calculations.
Many early models assumed constant temperatures of the flowing fluid as Edvardson et al (1962). Others used experimentally determined approximate correlations as Dowdle and Cobb (1975). A sophisticated analytical model was made by Boldizsár (1958) neglecting the thermal resistance of the multiple casing string. Raymond’s first numerical model (1969) has included the transient response of the flow for the initial short-time period.

The present model is an analytical approach, in which the unnecessary simplifications are avoided; a rigorous analytical treatment is applied as far as it is possible within reasonable limits. The temperature distribution is determined along the depth both in the drilling pipe and the annulus. Influencing factors: flow rate, well completion, elapsed time and geothermal conditions of the surrounding rock are investigated.

2. Formulation of the problem

The so-called forward circulation system is investigated, where the drilling fluid flows down in the drillpipe and back up in the annulus.

The thermal interaction between the drilling fluid and the formation is considered axisymmetric. In accordance to the system’s geometry a cylindrical coordinate system is chosen. Its z-axis is coaxial with the drillpipe, directing downward. The origin \( z = 0 \) is at the surface.

At the depth of \( z \) a suitable chosen control volume is taken in order to write the balance equation of the internal energy. The control volume is coaxial with the borehole. Its boundaries consist of a cylindrical surface of radius \( R_\infty \) and two horizontal parallel planes of distance \( dz \) between them. The radius \( R_\infty \) belongs to the location of the undisturbed formation temperature \( T_\infty \). The temperature difference between the formation and the drilling fluid induces a radial heat flux through several elements of the wellbore. The control volume is shown in Figure 1. It is convenient to separate it into four sub-systems: the flowing fluid in the drillpipe and the annulus, the well completion and the formation.

![Figure 1: Elementary control surface](image)
In the downward flow through the drillpipe the forced convection is the dominant mechanism of the heat transfer. In the upward flow through the annulus there is a twofold forced convection on both surfaces of the annulus. The well completion may be a completed section of the borehole, but it may be an open hole where the only thermal resistance is simply the forced convection between the formation and drilling fluid. In the formation radial conduction is the definitive phenomenon. This heat flux crossing the boundaries of the sub-systems must be the same on both sides.

Before balance equations are written, some simplifying assumptions can be taken. The drilling fluid is considered incompressible. The flow is steady and turbulent both through the drillpipe and the annulus. It is well known that the velocity and temperature distribution becomes more uniform over the pipe cross section as the Reynolds number increases, while the average velocity tends to the hypothetical velocity distribution of a perfect fluid. Thus the cross-sectional average velocities and temperatures can be used in the balance equations. The steep temperature change near the pipe wall in the thermal boundary layer is replaced by an abrupt temperature drop between the solid wall and the flowing fluid. The axial component of the conductive heat flux in the fluid is negligible. The temperature field of the formation is considered to axisymmetric around the borehole. The rate of change of the temperature in the fluid is substantially greater than in the surrounding rock. Thus the transient heat transfer process can be considered to a slow temperature change of the huge heat capacity rock mass, while the thermal response of the tiny fluid filament in the borehole follows it instantaneously.

Thereafter the balance equation of the internal energy can be written for the drillpipe as follows.

\[
\dot{m} c_{D} \frac{dT}{dz} = 2\pi R_{D} U_{D} (T_{A} - T_{D})
\]

Where \( \dot{m} \) is the mass flow rate of the fluid [kg/s], \( c \) is its specific heat capacity [J/kg °C]. \( U_{iv} \) is the overall heat transfer coefficient referring to the inner surface of the drillpipe [W/m² °C]. \( T_{D} \) and \( T_{A} \) are the flowing fluid temperatures in the drillpipe and the annulus. The other notations are clearly shown in Figure 1.

The internal energy balance for the upflowing fluid in the annulus is

\[
\dot{m} c_{A} \frac{dT}{dz} = 2\pi R_{B} U_{D} (T_{A} - T_{B}) + 2\pi R_{C} U_{C} (T_{B} - T_{A})
\]

where \( U_{ci} \) is the overall heat transfer coefficient referring to the inner surface of the casing. \( T_{B} \) is the temperature of the cement sheet at the borehole radius \( R_{B} \).

The heat fluxes at the boundary of the cement sheet and the surrounding rock are the same:

\[
2\pi R_{C} U_{C} (T_{B} - T_{A}) = 2\pi k_{R} \frac{T_{\infty} - T_{B}}{f(t)}
\]

where \( k_{R} \) is the heat conductivity of the rock [W/m °C]. \( T_{\infty} \) is the undisturbed temperature of the rock at the given depth:

\[
T_{\infty} = T_{0} + \gamma z
\]
$T_0$ is the surface temperature, $\gamma$ is the geothermal gradient [°C/m], $f(t)$ is the so-called transient heat conduction function, depending on the Fourier number and the quantity $\frac{R_C U_C}{k_R}$.

3. Solution

The balance equation (1) can be slightly modified:

$$\frac{dT_D}{dz} = \frac{T_A - T_D}{B}$$

where the quantity

$$B = \frac{\dot{m}c}{2\pi R_{Di} U_{Di}}$$

doesn’t depends on the depth $z$.

Balance equations (1) and (2) are added, thus we get:

$$\dot{m}c \left( \frac{dT_A}{dz} + \frac{dT_D}{dz} \right) = 2\pi R_C U_C \left( T_B - T_A \right)$$

The temperature difference $T_B - T_A$ can be expressed from (7)

$$T_B - T_A = \frac{\dot{m}c}{2\pi R_C U_C} \left( \frac{dT_A}{dz} + \frac{dT_D}{dz} \right)$$

Similarly $T_\infty - T_B$ is obtained from Eq. (3):

$$T_\infty - T_B = \frac{R_C U_C f(t)}{k_R} \left( T_B - T_A \right)$$

The sum of Eq. (8) and Eq. (9) obtains.

$$T_\infty - T_A = \left( \frac{\dot{m}c}{2\pi R_C U_C} + \frac{R_C U_C f(t)}{k_R} \right) \left( \frac{dT_A}{dz} + \frac{dT_D}{dz} \right)$$

After some simplification we get the expression

$$\frac{dT_A}{dz} + \frac{dT_D}{dz} = \frac{T_\infty - T_A}{A}$$
in which the so-called performance state coefficient

\[ A = \frac{\min\left(k_R + R_{C_i} U_{C_i} f \right)}{2\pi R_{C_i} U_{C_i} k_R} \]  

seems to be independent of the depth \( z \). Nevertheless \( A \) depends linearly on the circulating mass flow rate and the specific heat capacity of the drilling fluid. Thus at a given state of performance \( A \) can be considered constant. In the case of constant \( A \) the equation system is linear and an analytic solution can be obtained relatively easily. However \( A \) has a weak dependence on depth, temperature and time.

In accordance to the well completion \( R_{C_i} \) and \( U_{C_i} \) changes in different depth intervals. Heat conductivity \( k_R \) can be replaced by its depth-averaged value. Heat transfer coefficient between flowing fluid and pipe wall depends on the viscosity, i.e. the temperature. In fluid-filled annular sections natural convection occurs. Its heat transfer coefficient depends on the temperature both directly and because of the viscosity-dependence indirectly too. The transient heat conduction function \( f \) depends on the Fourier-number, i.e. the time and the well completion. At a given time it can be taken to constant, thus we obtain different \( f \) values for different time-steps. Thus, however Eq. (11) has a slightly non-linear character, simple depth averaged material properties are suitable to determine temperature distributions.

Finally the differential equation system from pure mathematical point of view is

\[ A \frac{dT_A}{dz} + A \frac{dT_D}{dz} = T_{\infty} - T_A \]  

and

\[ B \frac{dT_D}{dz} + T_D = T_A \]  

Differentiating Eq. (14) by \( z \), after substitution we obtain a second-order, linear, inhomogeneous differential equation with constant coefficient for \( T_s \):

\[ A B \frac{d^2T_D}{dz^2} - B \frac{dT_D}{dz} - T_D + T_0 + \gamma z = 0 \]  

The homogeneous differential equation belonging to it is:

\[ A B \frac{d^2T_D}{dz^2} - B \frac{dT_D}{dz} - T_D = 0 \]  

Its characteristic equation is

\[ A B \lambda^2 - B \lambda - 1 = 0 \]
The roots of the characteristic equation are:

$$\lambda_1 = \frac{1}{2A} \left( 1 + \sqrt{1 + \frac{4A}{B}} \right)$$ \hspace{1cm} (18)

and

$$\lambda_2 = \frac{1}{2A} \left( 1 - \sqrt{1 + \frac{4A}{B}} \right)$$ \hspace{1cm} (19)

Since both A and B are real, the solution of the homogeneous differential equation is obtained as

$$T_{D_{\text{hom}}} = C_1 e^{\lambda_1 z} + C_2 e^{\lambda_2 z}$$ \hspace{1cm} (20)

Since the right-hand side of the Eq. (15) is linear, a particular solution of it is looking for also in linear form:

$$T_{D_{\text{inh}}} = \alpha + \beta z$$ \hspace{1cm} (21)

Substituting it into the Eq. (15), we get

$$T_{D_{\text{inh}}} = B\gamma - T_0 - \gamma z$$ \hspace{1cm} (22)

Thus the general solution of Eq. (15) is the sum of Eqs. (20) and (22):

$$T_D = C_1 e^{\lambda_1 z} + C_2 e^{\lambda_2 z} + T_0 + \gamma z - B\gamma$$ \hspace{1cm} (23)

The temperature distribution of the annular flow can be determined substituting Eq. (23) into Eq. (14). Thus we get:

$$T_A = C_1 (1 + B\lambda_1) e^{\lambda_1 z} + C_2 (1 + B\lambda_2) e^{\lambda_2 z} + T_0 + \gamma z$$ \hspace{1cm} (24)

The constant coefficients $C_1$ and $C_2$ can be determined satisfying the following boundary conditions: If $z = 0$, $T_D = T_e$. That is the drilling fluid temperature at the entrance in the drillpipe on the surface is $T_e$. The other is that the bottomhole temperatures both in the drillpipe and the annulus are the same. If $z = H$, $T_D(H) = T_A(H)$. Thus the following linear algebraic equation system is obtained

$$C_1 + C_2 = T_e - T_0 + B\gamma$$ \hspace{1cm} (25)

and

$$C_1\lambda_1 e^{\lambda_1 H} + C_2\lambda_2 e^{\lambda_2 H} = -B\gamma$$ \hspace{1cm} (26)
The roots of this equation system are

\[ C_1 = \frac{D_1}{D}; \quad C_2 = \frac{D_2}{D} \]  

(27)

where

\[ D = \lambda_2 e^{\lambda_2 H} - \lambda_1 e^{\lambda_1 H} \]

\[ D_1 = \lambda_2 (T_e - T_0 + B \gamma) e^{\lambda_2 H} + \gamma \]

\[ D_2 = -\lambda_1 (T_e - T_0 + B \gamma) e^{\lambda_2 H} - \gamma \]

(28)

Temperature distributions \( T_{D}(z) \) and \( T_{A}(z) \) can be determined by Eqs. (23) and (24). In order to calculate these it is necessary to evaluate the constants \( A \) and \( B \). Both constants depend on the data of borehole geometry, drilling fluid properties, surrounding rock parameters and the data of performance. These are the following:

- borehole depth, drillpipe outer and inner diameters, drill bit size, the instantaneous completion of the borehole,
- entrance mud temperature, density, viscosity, specific heat capacity and heat conductivity of the mud,
- average density, heat conductivity, specific heat capacity of the rock, surface earth temperature, geothermal gradient,
- circulation mass flow rate, mud temperature at the entrance, elapsed time during drilling operation.

Knowing these data the overall heat transfer coefficients \( U_{Di} \) and \( U_{Ci} \), the transient heat conduction function \( f(t) \) can be calculated in the well-known manner (WILLHITE, 1967). The transient heat transfer function has different values belonging successive steps of elapsed time.

Using the evaluated constants \( A \) and \( B \) temperature distributions is obtained from Eqs. (23) and (24). It is obvious that many independent variables influence temperature distributions in the borehole. This can be followed in the calculated temperature distribution diagrams.

4. Conclusions

Considerable observations can be made examining the calculated temperature distribution functions.

The temperature of the downward flow in the drillpipe increases monotonically. Bottomhole temperatures decrease as the mass flow rates increase. Temperature gradient of the downward flow at the bottomhole equals zero. These observations are demonstrated in Figure 2.

As the fluid turns upward at the bottomhole temperature increases until attains its maximum. The location of this maximum is obtained at hardly different depths as it is shown in Figure 3. It is remarkable that the annular temperature distribution differs greatly from the undisturbed natural geothermic temperature.
Figure 2: The temperature distribution in the drillpipe

Figure 3: The temperature distribution in the annulus
Above this depth the temperature of the upflowing fluid decreases and converges to the downflowing fluid temperature as shown in Figure 4.

**Figure 4: The temperature distribution in the drillpipe and in the annulus**

**Figure 5: The effect of the injection temperature on the temperature distributions**
Both the downflowing and the upflowing fluid temperature depend on the mass flow rate and the specific heat capacity of the circulating drilling fluid. As mass flow rate increases the fluid temperature decreases since the performance coefficients A and B are linear functions of the mass flow rate.

Bottomhole temperature is also influenced by the mass flow rate. Entrance temperature of the drilling fluid influences the bottomhole temperature especially at high mass flow rates. The change of the bottomhole temperature can be seen in Figure 5.

The introduced observations based on results of our mathematical model seem to have a general validity.

5. Summary

An analytical mathematical model is presented to predict the temperature distribution of the circulating drilling fluid both in the drillpipe and in the annulus. Calculations are referred to forward circulation. Many independent variables influence temperature distribution in the drillpipe and the annulus. Some of these are constant like rock properties geothermal gradient, borehole geometry, etc. Others change as performance parameters vary, e.g. mass flow rate, entrance fluid temperature, the elapsed time. The knowledge of the temperature distribution in the circulating drilling fluid can be applied in the design of different drilling operations.

LEGENDS

\[\begin{align*}
A & \quad \text{borehole performance state coefficient} \\
B & \quad \text{drillpipe performance state coefficient} \\
c & \quad \text{specific heat capacity} \\
C_1, C_2 & \quad \text{constants of integration} \\
D & \quad \text{auxiliary variable} \\
D_1, D_2 & \quad \text{auxiliary variables} \\
f & \quad \text{transient heat conduction function} \\
H & \quad \text{bottomhole depth} \\
k & \quad \text{heat conductivity} \\
\dot{m} & \quad \text{mass flow rate} \\
R_{Di} & \quad \text{inner radius of the drillpipe} \\
R_{Ci} & \quad \text{inner radius of the casing} \\
R_B & \quad \text{radius of the borehole} \\
R_e & \quad \text{radius of the undisturbed temperature field} \\
T & \quad \text{temperature} \\
T_A & \quad \text{drilling fluid temperature in the annulus} \\
T_D & \quad \text{drilling fluid temperature in the drillpipe} \\
T_0 & \quad \text{surface temperature} \\
T_e & \quad \text{drilling fluid temperature at the entrance} \\
T_{\infty} & \quad \text{undisturbed rock temperature} \\
U_{Di} & \quad \text{overall heat transfer coefficient of the drillpipe}
\end{align*}\]
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